Heterodyne polarimetry

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Greta Guidi - Postdoctoral researcher, IPAG Grenoble

Outline

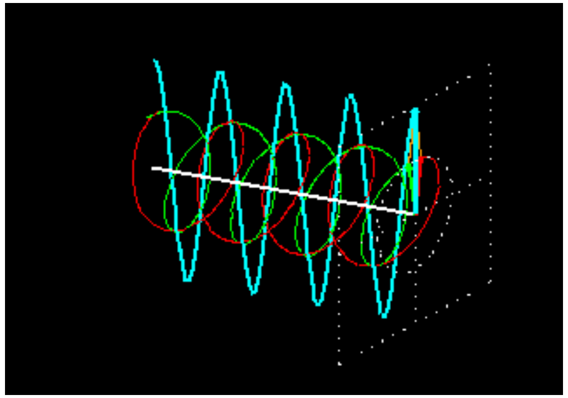
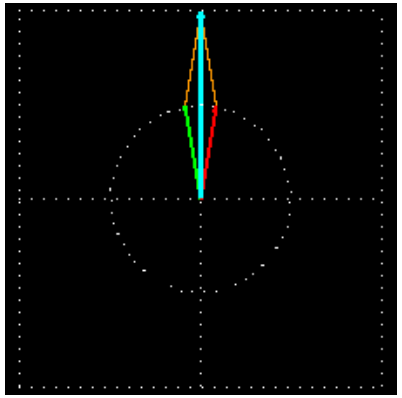
- Some basics:
    - Circular/linear basis
    - Stokes Parameters
- Calibration
- Polarization @NOEMA 3mm
- Polarization calibrators archive

Disclaimer: slight bias towards interferometry

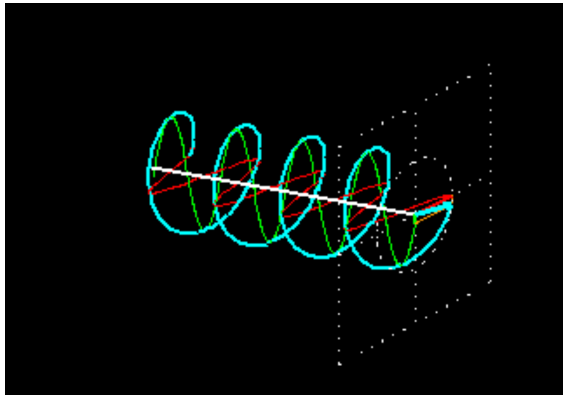
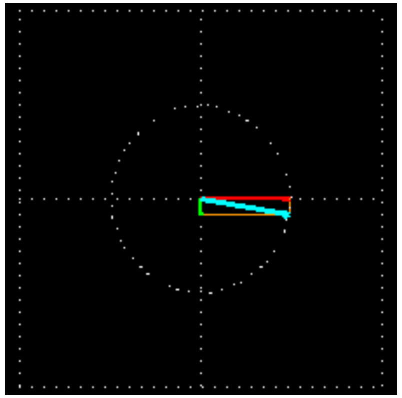
IRAM/NOEMA

Circular and linear basis

(Sub-)Milllimeter antennas: measure orthogonal components of the Electric field.

Linearly polarised wave can be obtained by a superposition of a R and L circularly polarised waves

Circularly polarised wave can be obtained by a superposition of 2 linearly polarised waves with a phase difference of 90°

Most heterodyne receivers record signal in a linear basis.
A polarizer is sometimes used (e.g. a quarter-wave plate) to convert linear to circular

Polarimetry with circular and linear basis

Advantages and disadvantages

$E_X\,, E_Y$

Linear

. Practical: antenna polarizers are naturally linear
. Calibration is more complex (see also mixed-basis calibration, e.g. VLBI)
. Better for measuring circular polarization

E_R, E_L

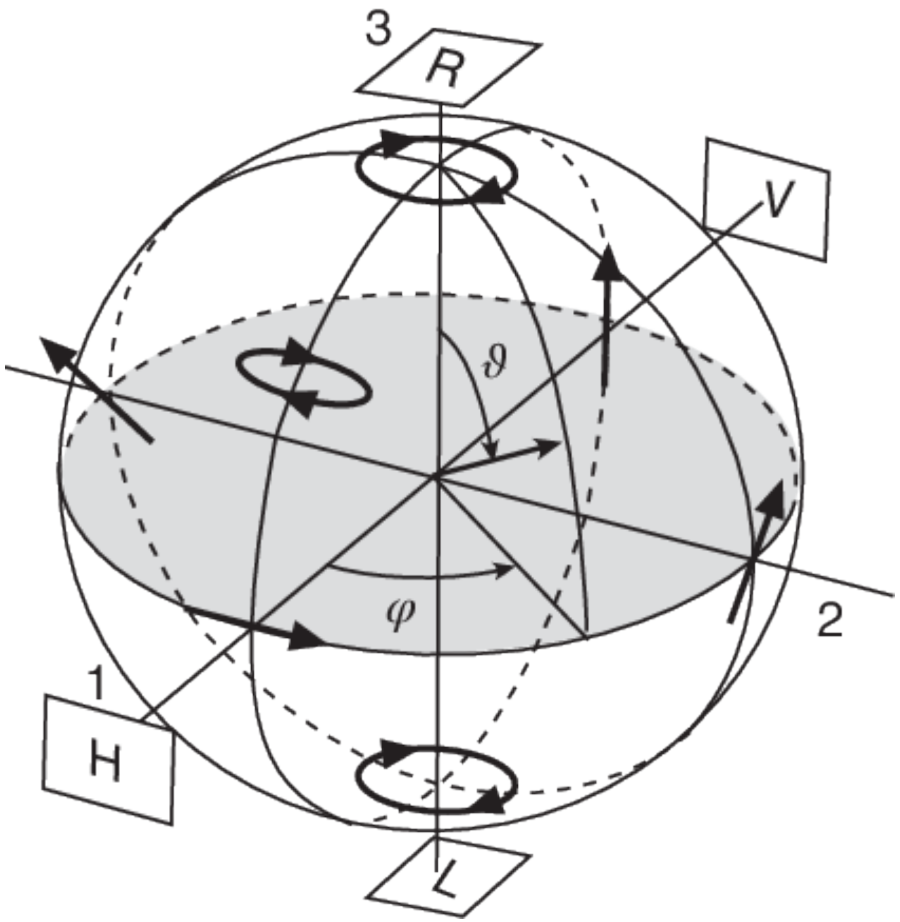
Circular

. Need additional elements for conversion liner-circular:
- more cross-talk (leakages up to 5%)
- sub-optimal performances in wide bands

. Calibration is easier
. Better for measuring linear polarization

Stokes parameters

We need 4 parameters to characterize the polarisation state of an EM wave: I, Q, U, V



$$I = A_X^2 + A_Y^2 = A_R^2 + A_L^2$$

$$Q = A_X^2 - A_Y^2 = 2A_R A_L \cos\delta_{RL}$$

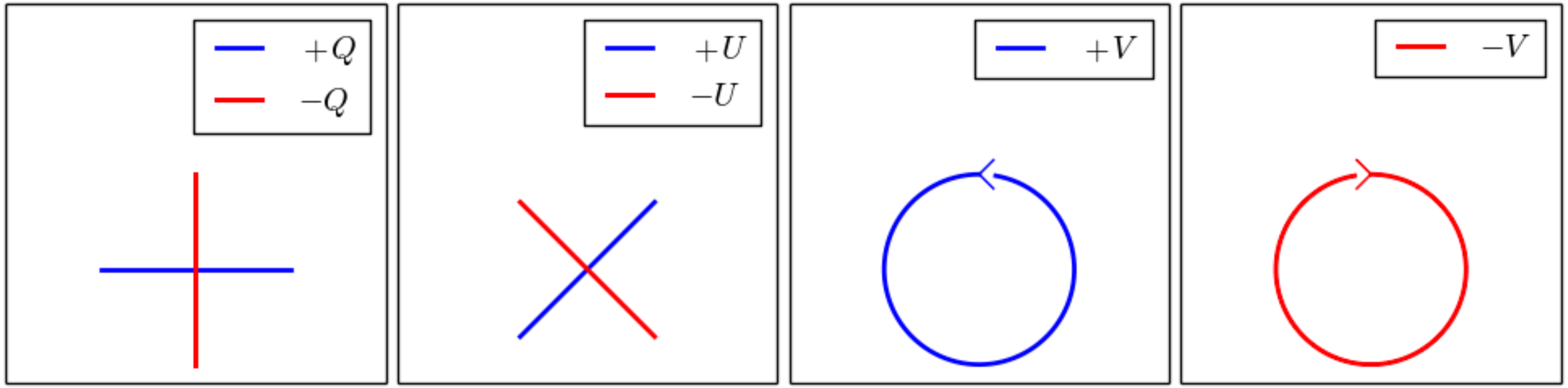
$$U = 2A_X A_Y \cos\delta_{XY} = 2A_R A_L \sin\delta_{RL}$$

$$V = 2A_X A_Y \sin\delta_{XY} = A_R^2 - A_L^2$$

δ = phase difference

A = amplitude of the Electric field

Stokes parameters

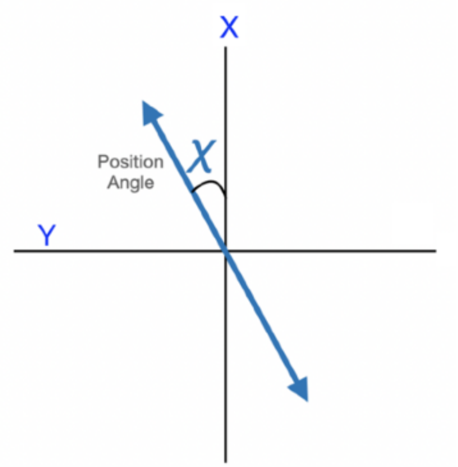


Degree of linear polarisation $$m_l = \frac{\sqrt{Q^2 + U^2}}{I}$$

Degree of circular polarisation $$m_c = \frac{V}{I}$$

Polarisation angle $$EVPA = \frac{1}{2} arctan\left(\frac{U}{Q}\right)$$

Unpolarised intensity $$I_u = \sqrt{I^2 - Q^2 - U^2 - V^2}$$



* Note that in the pulsar community the convention can be different, i.e. V>0 is left-handed polarization ("PSR/IEEE" convention)

Calibration

General matrix calibration

$$s_{\mathrm{obs},i} = J_i s_{\mathrm{true},i} = B_i G_i D_i P_i T_i F_i s_i \qquad s = \begin{pmatrix} s_p \\ s_q \end{pmatrix}$$

Bandpass (B_i), Gain (G_i), Leakages (D_i), Parallactic angle (P_i), Troposphere (T_i), Ionispheric Faraday rotation (F_i)

We can imagine each antenna's received signal corrupted from right to left by the different factors

In interferometry: $$V_{ij} = J_i s_i (s_j)^\dagger J_j^\dagger$$

$$V_l = \begin{pmatrix} V_{XX} \\ V_{XY} \\ V_{YX} \\ V_{YY} \end{pmatrix} \quad V_c = \begin{pmatrix} V_{RR} \\ V_{RL} \\ V_{LR} \\ V_{LL} \end{pmatrix}$$

$$V_{RR} = \mathcal{I} + \mathcal{V} \qquad V_{XX} = \mathcal{I} + \mathcal{Q}$$

$$V_{RL} = \mathcal{Q} + i\mathcal{U} \qquad V_{XY} = \mathcal{U} + i\mathcal{V}$$

$$V_{LR} = \mathcal{Q} - i\mathcal{U} \qquad V_{YX} = \mathcal{U} - i\mathcal{V}$$

$$V_{LL} = \mathcal{I} - \mathcal{V} \qquad V_{YY} = \mathcal{I} - \mathcal{Q}$$

Calibration - parallactic angle correction

Alt-azimuthal mounting: the telescope feeds rotate with respect to the sky plane, with a characteristic angle called Parallactic Angle

$$\tan\psi_p = \frac{\cos\mathcal{L}\sin\mathcal{H}}{\sin\mathcal{L}\cos\delta - \cos\mathcal{L}\sin\delta\sin\mathcal{H}}$$

If $\Psi_i = \Psi$ for all antennas (small array)

$$P_l = \begin{pmatrix} cos\psi & sin\psi \\ -sin\psi & cos\psi \end{pmatrix} \qquad P_c = \begin{pmatrix} e^{-i\psi} & 0 \\ 0 & e^{i\psi} \end{pmatrix}$$

$$V_{XX} = \mathcal{I} + (\mathcal{Q}\cos 2\psi + \mathcal{U}\sin 2\psi) = \mathcal{I} + \mathcal{Q}_\psi$$

$$V_{XY} = (-\mathcal{Q}\sin 2\psi + \mathcal{U}\cos 2\psi) + i\mathcal{V} = \mathcal{U}_\psi + i\mathcal{V}$$

$$V_{YX} = (-\mathcal{Q}\sin 2\psi + \mathcal{U}\cos 2\psi) - i\mathcal{V} = \mathcal{U}_\psi - i\mathcal{V}$$

$$V_{YY} = \mathcal{I} - (\mathcal{Q}\cos 2\psi + \mathcal{U}\sin 2\psi) = \mathcal{I} - \mathcal{Q}_\psi$$

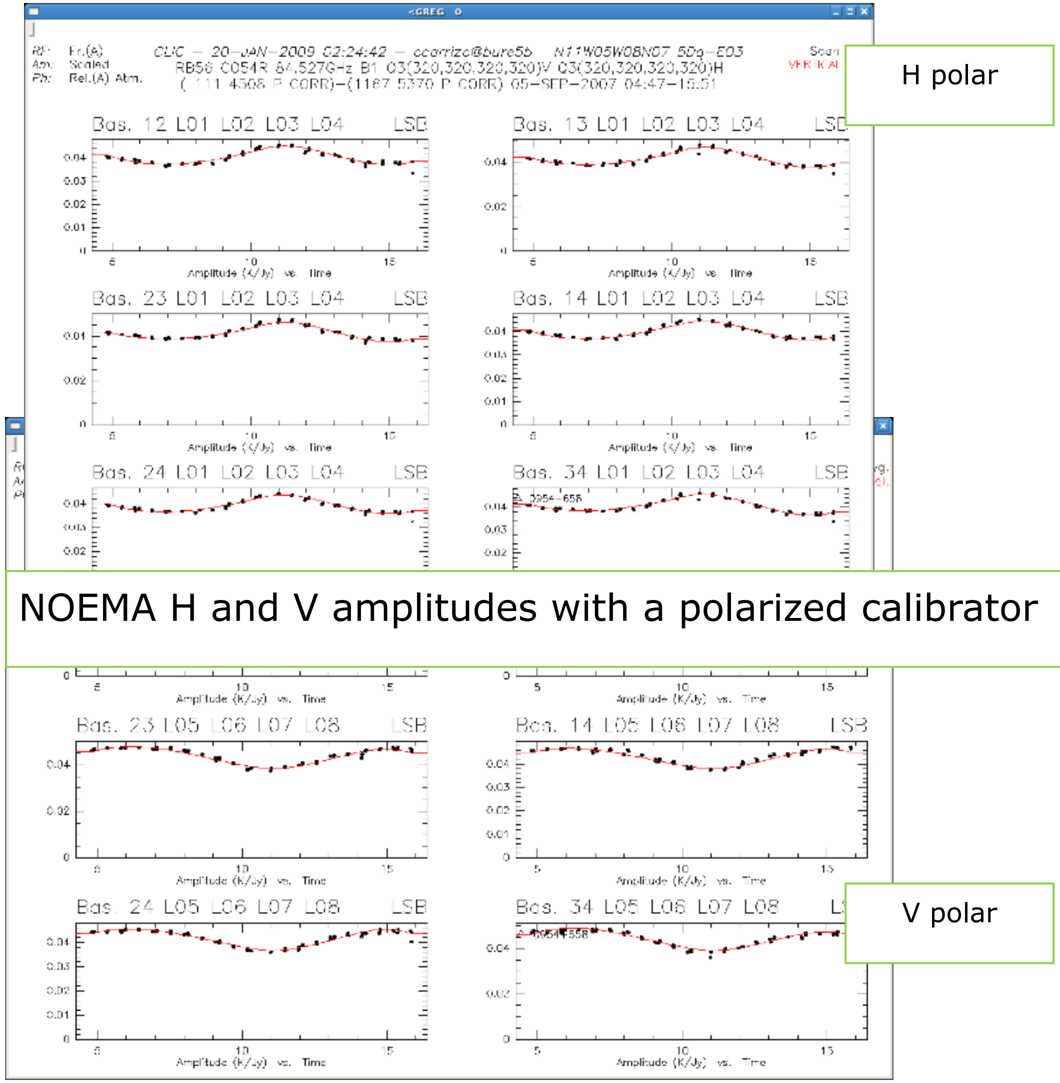
$$V_{RR} = \mathcal{I} + \mathcal{V}$$

$$V_{RL} = (\mathcal{Q} + i\mathcal{U})\, e^{-i2\psi}$$

$$V_{LR} = (\mathcal{Q} - i\mathcal{U})\, e^{+i2\psi}$$

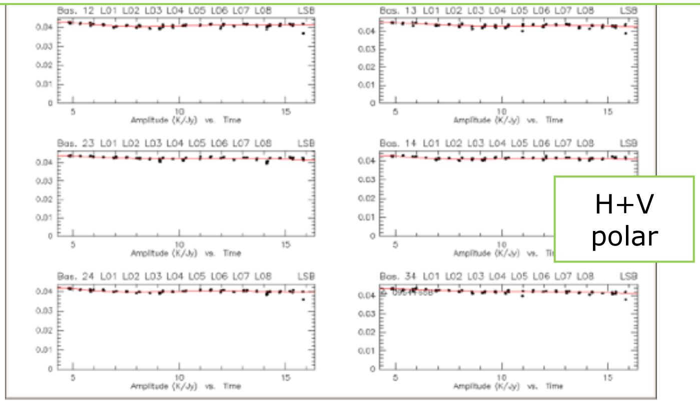
$$V_{LL} = \mathcal{I} - \mathcal{V}$$

Calibration - parallactic angle correction



NOEMA H and V amplitudes with a polarized calibrator

Below polarizations are averaged to perform the amplitude calibration



In the linear basis, also parallel hands vary with time/parallactic angle -> affects standard calibration as well

Calibration - parallactic angle correction

VLBI: different parallactic angles between the stations

VLBI uses circular basis (parallel hands not affected by PA rotation), but there are cases of mixed polarization when a station has linear receivers
(NOEMA uses quarter-wave plates to convert linear to circular basis)

Phased arrays or single stations with linear receivers: blind conversion linear to circular is not possible (gain errors in X/Y translate into gain-like and leakage-like effects in R/L)

→ ALMA: *PolConvert tool* (Marti-Vidal + 2016), reads CASA calibration tables

→ Single stations: derive gain ratio between the 2 polarization, assuming negligible leakages and no circular polarization of the source (sort of self-calibration)
PolConvertST script to be used in AIPS (no calibration applied before conversion)

Leakages (instrumental polarization)

Origins of the leakages:
– Finite impurities in polarizers
– Reflections that return in opposite polarization: standing waves
– Asymmetry in optics

Dependent on frequency!

$$D = \begin{pmatrix} 0 & d_p \\ d_q & 0 \end{pmatrix}$$

Properties: orthogonality

$$d_{p,i} + d^*_{q,i} = 0$$

In the linear basis (for d << 1.0):
– Real(d) = linear polarization orientation error
– Imag(d) = ellipticity error

Polarisation in interferometry: calibration

In the linear approximation:

$$
\begin{aligned}
V_{ij}(HH) &= g_{iH} g_{jH}^{*} \left[I - Q\cos(2\chi) - U\sin(2\chi) \right] \\
V_{ij}(HV) &= g_{iH} g_{jV}^{*} \left[(d_{iY} + d_{jX}^{*}) I - Q\sin(2\chi) + U\cos(2\chi) + \mathrm{i}V \right] \\
V_{ij}(VH) &= g_{iV} g_{jH}^{*} \left[(d_{iX} + d_{jY}^{*}) I - Q\sin(2\chi) + U\cos(2\chi) - \mathrm{i}V \right] \\
V_{ij}(VV) &= g_{iV} g_{jV}^{*} \left[I + Q\cos(2\chi) + U\sin(2\chi) \right]
\end{aligned}
$$

Leakages

Additional effects to calibrate for:

. Cross-phase (phase difference between H and V)

. Cross-delay (delay difference between H and V)

. Amplitude offset between different polarizer channels (pathways)

Polarisation in interferometry: calibration

$$
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V_{ij}(HH) &= g_{iH} g^*_{jH} \left[I - Q\cos(2\chi) - U\sin(2\chi)\right] \\
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V_{ij}(VV) &= g_{iV} g^*_{jV} \left[I + Q\cos(2\chi) + U\sin(2\chi)\right]
\end{aligned}
$$

Leakages

. About ~1% of the total intensity (typically comparable to Q and U!)

. Measured (and corrected for) in the center -> the calibration accuracy decreases with distance from the center

ALMA: leakages ~1% computed in each channel

Min detectable linear polarization of 0.1% for compact sources (within 1/3 of the PB)

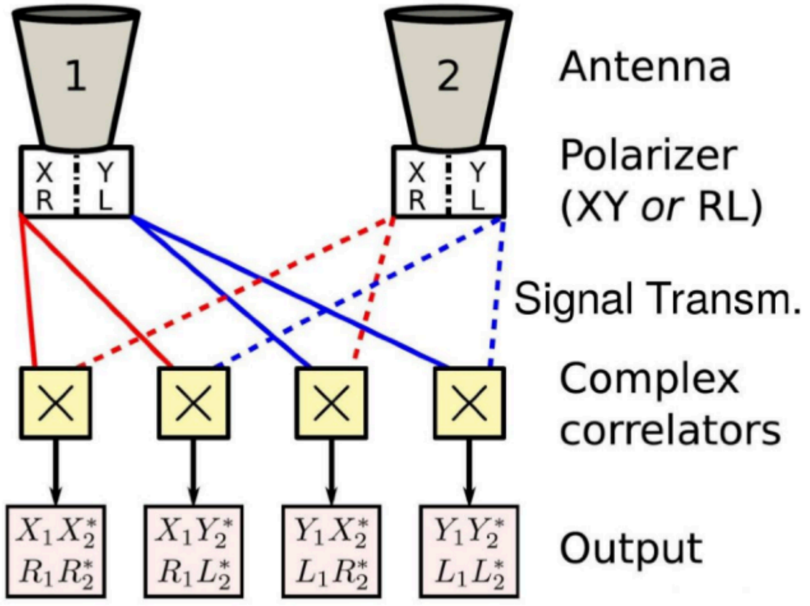
Min detectable circular polarization of 1.8% (within 1/10 of the PB)

Accuracy of polarization angle 1% within 1/3 of the PB, IF enough signal-to-noise!

PA coverage of 60 deg per each track (3-4 hours)

Polarisation @NOEMA

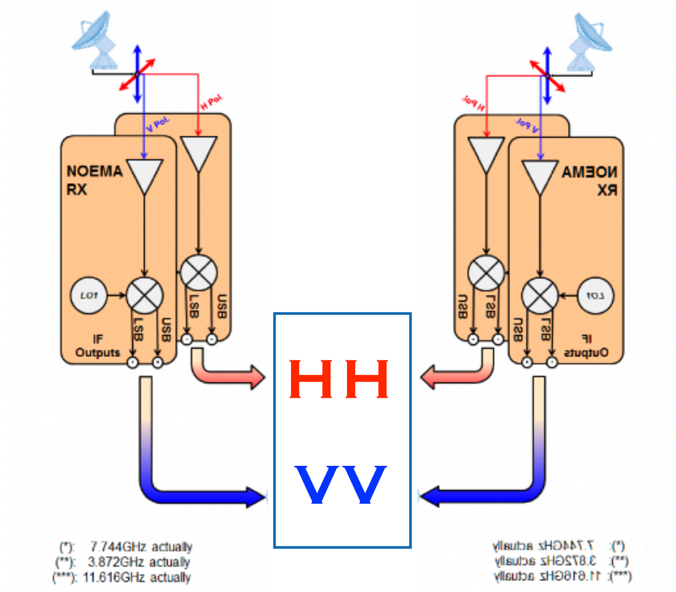
ALMA: linear feeds (X,Y)
+ full stokes correlator



Single polarisation: XX
Dual polarisation: XX, YY
Full polarisation: XX, YY, XY, YX

NOEMA: linear feeds (H,V), no full stokes correlator (cannot process the 4 products together)

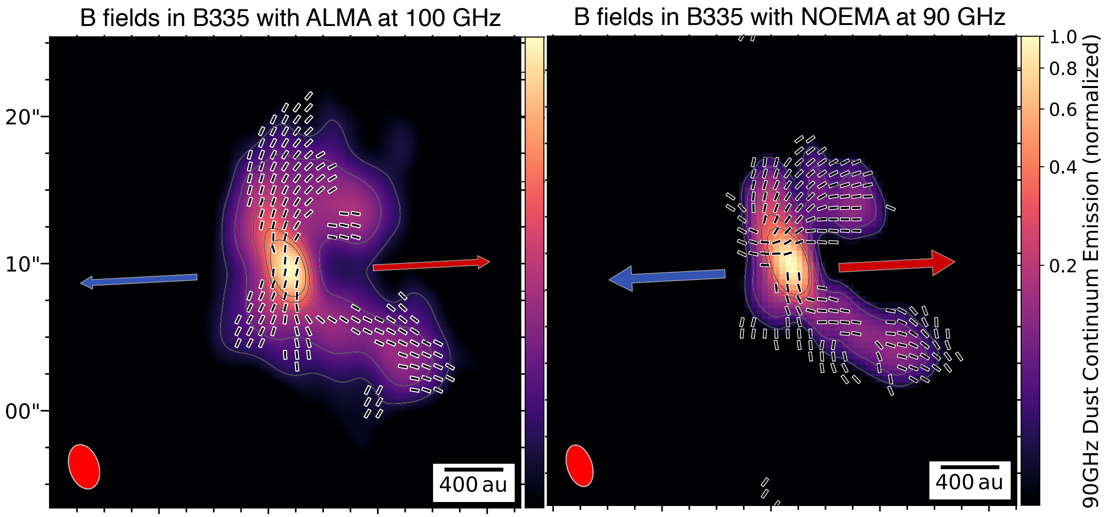
But H and V can be switched in the receiver cabin



Polarisation @NOEMA

Sponsored by ECOGAL, commissioning/SV from 2022. Kickoff meeting Nov 2023.

ENYGMA Large Program (PI Maury/Testi)
Currently observed 11 tracks, about 25 sources (Class0 YSO)



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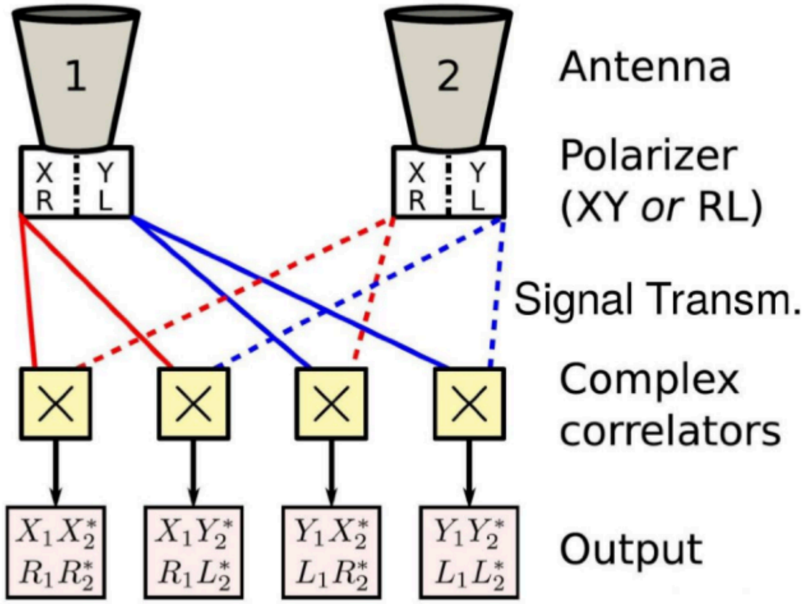

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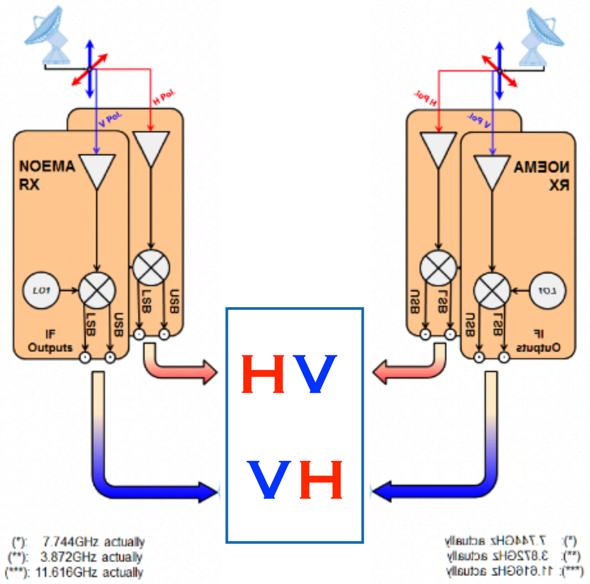
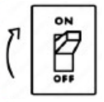




Switch crossed

No full Stokes correlator: observing strategies

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12
scan 1	1	1	1	1	1	1	1	1	1	1	1	1
scan 2	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1
scan 3	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1
scan 4	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1
scan 5	1	1	1	-1	-1	-1	-1	1	1	1	1	-1
scan 6	-1	1	-1	-1	1	-1	1	1	-1	1	-1	-1
scan 7	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1
scan 8	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1
scan 9	1	1	1	1	1	1	1	-1	-1	-1	-1	-1
scan 10	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1
scan 11	1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1
scan 12	-1	-1	1	1	-1	-1	1	-1	1	1	-1	-1
scan 13	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1
scan 14	-1	1	-1	-1	1	-1	1	-1	1	-1	1	1
scan 15	1	-1	-1	-1	-1	1	1	-1	-1	1	1	1
scan 16	-1	-1	1	-1	1	1	-1	-1	1	1	-1	1

NOEMA polarisation mode: strategy

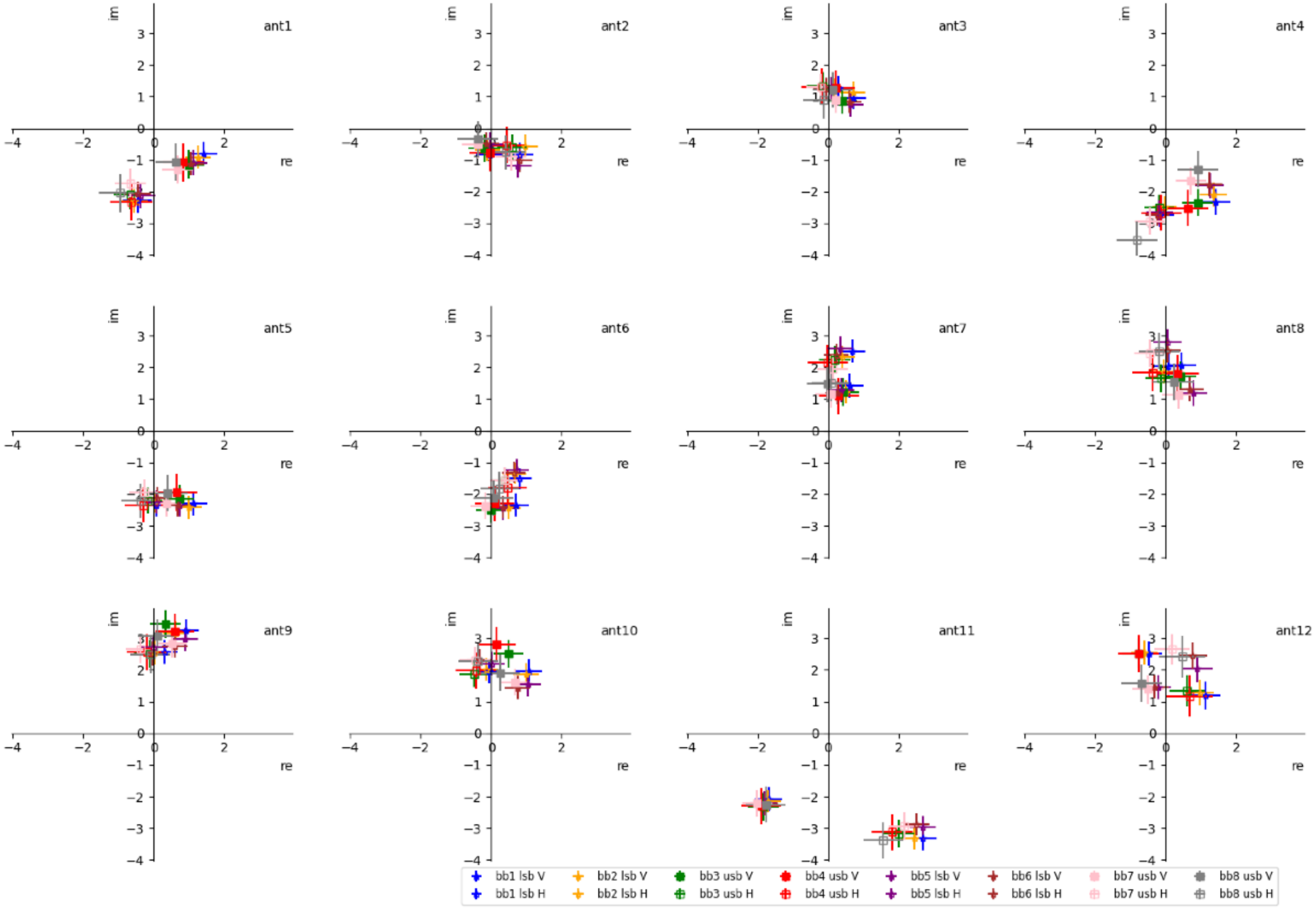
1 switch crossed (on)
-1 switch direct (off)

Walsh sequence of 16 states allowing to measure the four correlation products (HH, VV, HV, VH) on every baseline exactly 4 times.

→ Generates overheads

Scan cannot be too long because of time smearing

Estimated leakages at NOEMA



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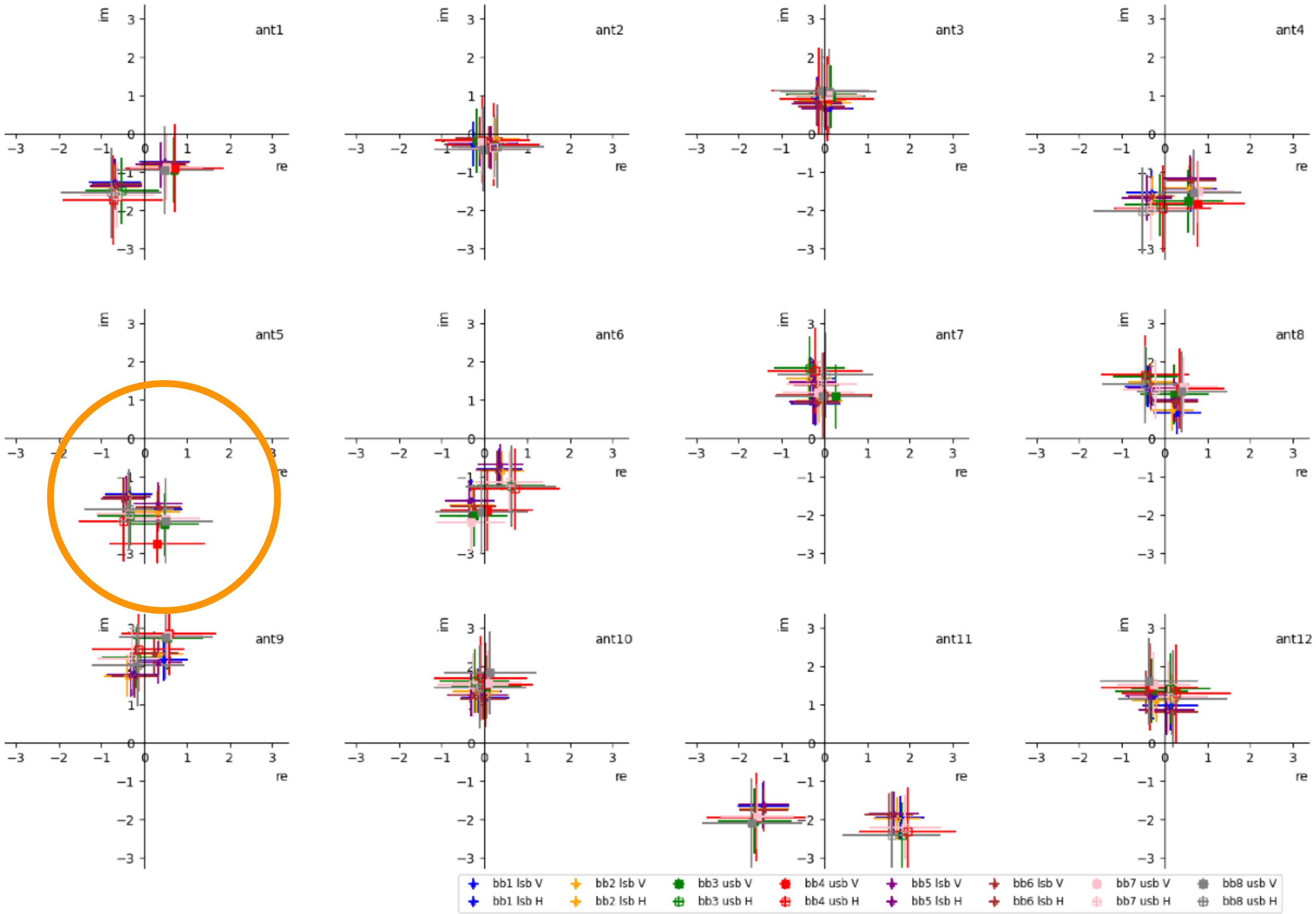
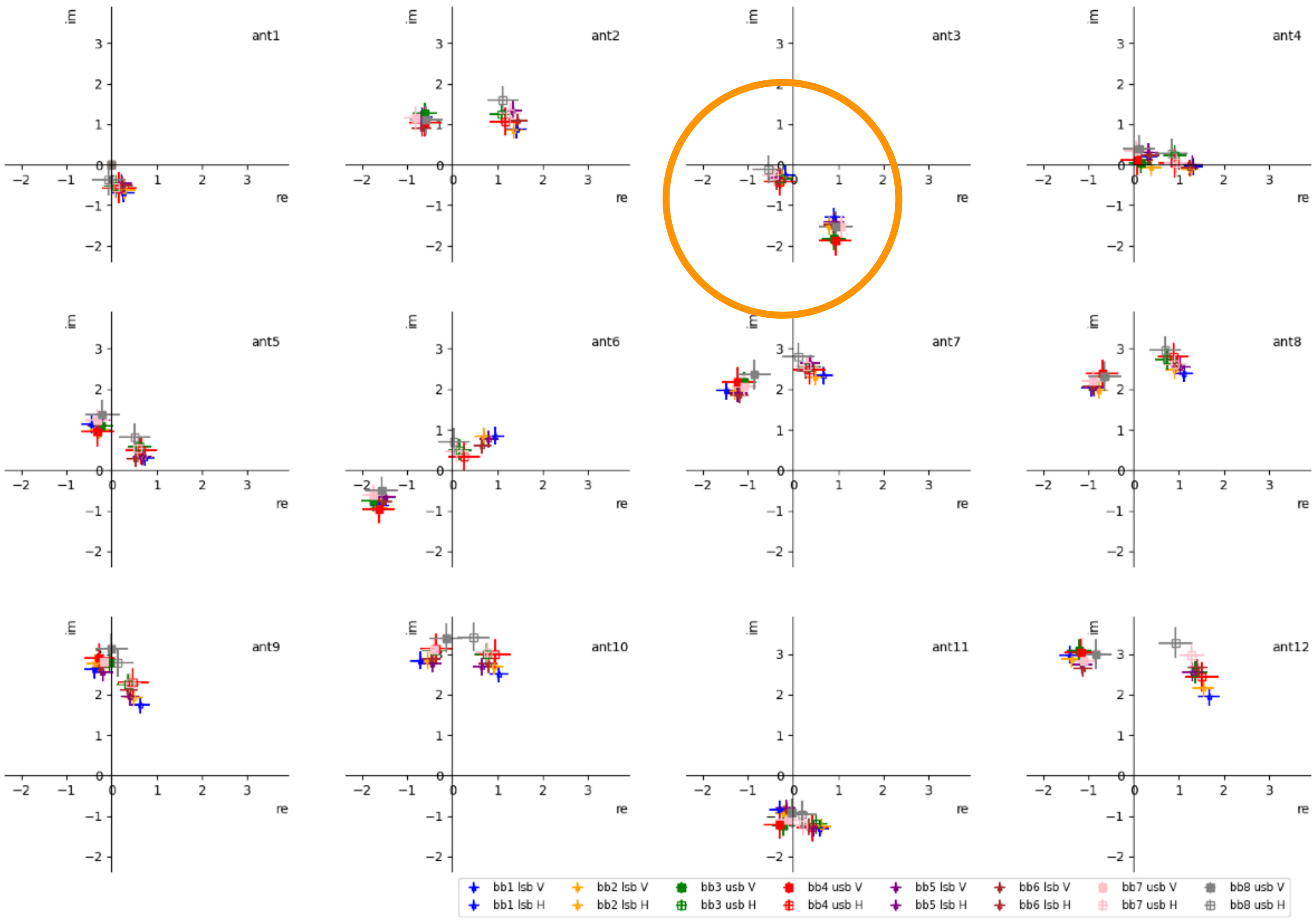


- ~ Consistent across different tracks
- Indications of orthogonality
- Correlations with changes in frontend elements (receivers swap)

Jan 2024

Estimated leakages at NOEMA



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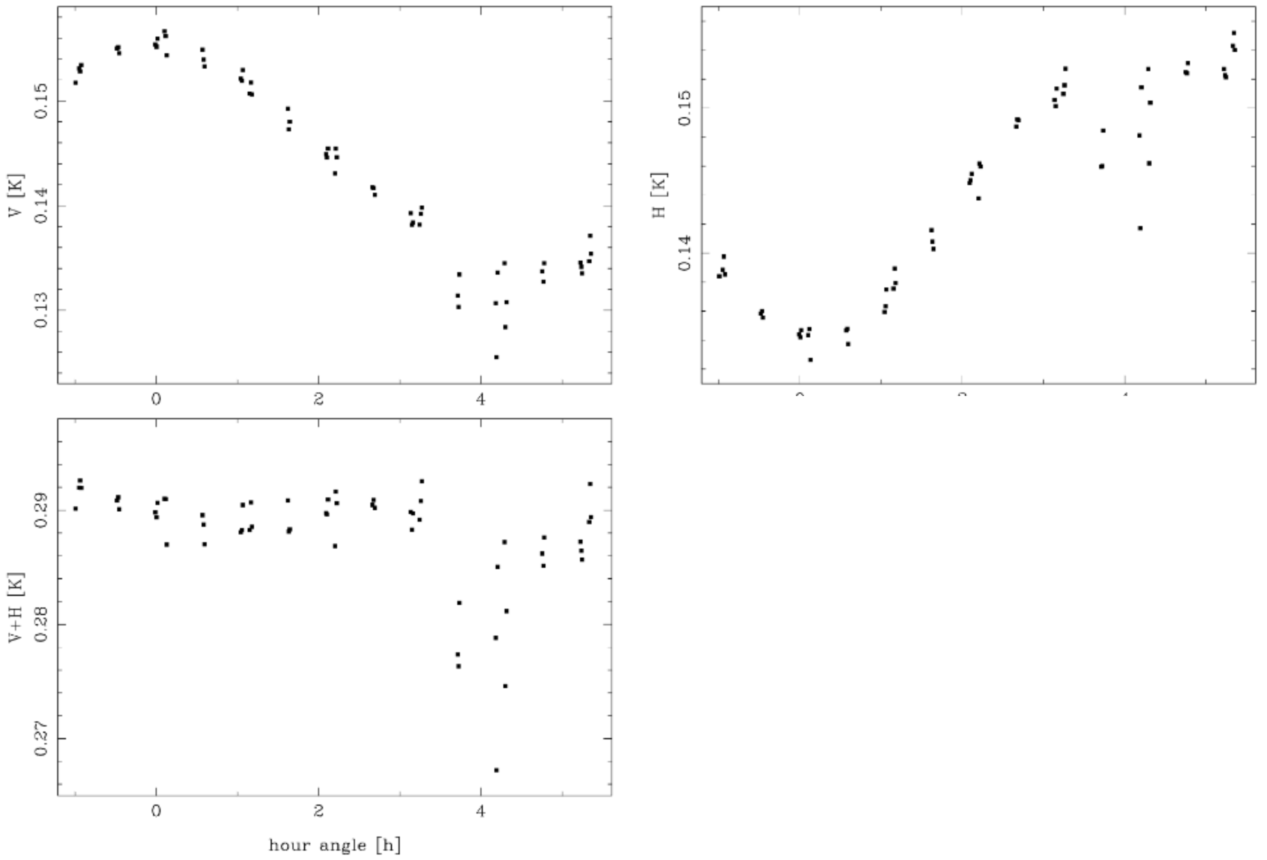
Jan 2025

Polarisation with parallel products only?

Yes we can (for point sources)



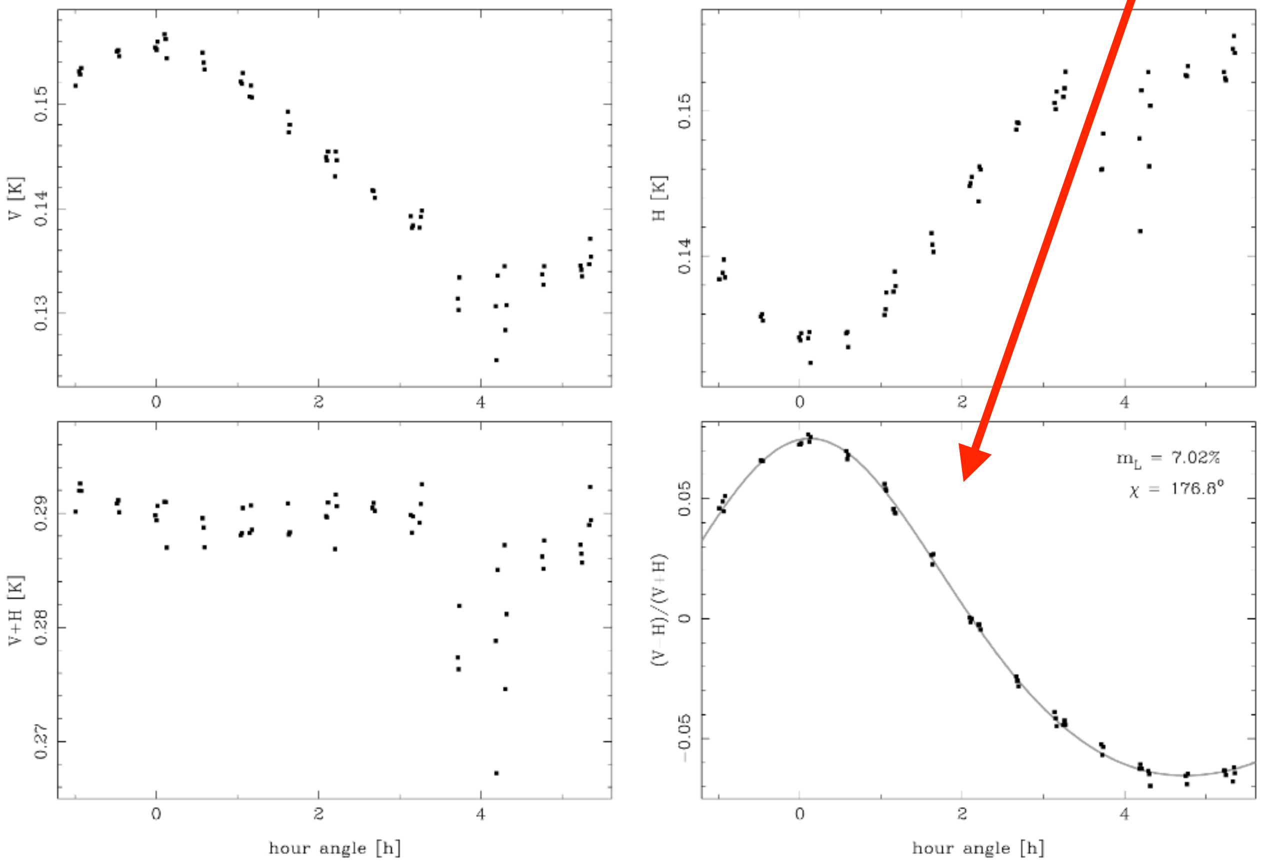
$$I = \frac{1}{2}\left[V_{HH} + V_{VV}\right]$$

Trippe et al. 2010

Polarisation with parallel products only?

Yes we can (for point sources)

$$\frac{VV-HH}{HH+VV} = \frac{Q}{I}\cos(2\chi) + \frac{U}{I}\sin(2\chi) + \epsilon$$



We can derive
. degree of linear polarization
. polarisation angle

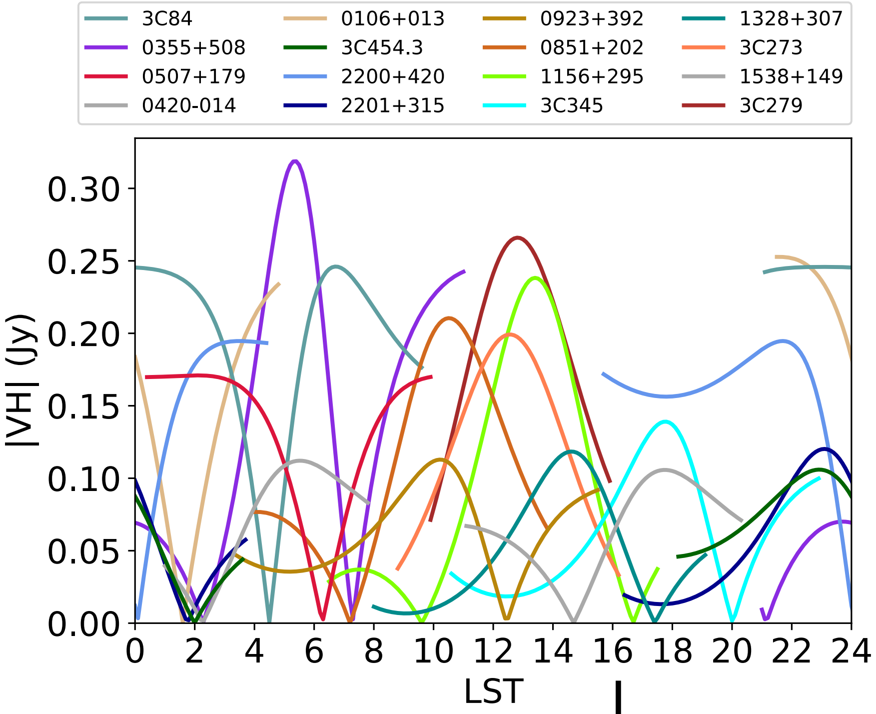
Independent from gain variations

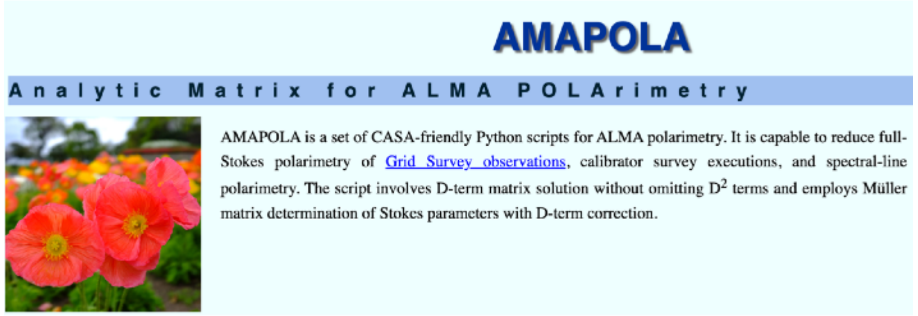
Trippe et al. 2010

* Note that this is for unresolved calibrators where Q and U are real

Polarisation with parallel products only?

Yes we can (for point sources)





https://www.alma.cl/~skameno/AMAPOLA/

XY YX correlation for ALMA calibrators

$$
\begin{aligned}
V_{ij}(HV) &= g_{iH} g_{jV}^{*} \left[(d_{iY} + d_{jX}^{*}) I - Q \sin(2\chi) + U \cos(2\chi) + \mathrm{i}V \right] \\
V_{ij}(VH) &= g_{iV} g_{jH}^{*} \left[(d_{iX} + d_{jY}^{*}) I - Q \sin(2\chi) + U \cos(2\chi) - \mathrm{i}V \right]
\end{aligned}
$$

We can estimate VH / HV as function of LST from unpolarized (standard) observations of sufficiently long tracks (≳ 2hr)

(necessary to plan optimally the observing strategy, choosing calibrators…)

Instrumental effects on polarization

- Reflections: turn RCP into LCP
- Curvature of surfaces
    - introduce cross-polarization
    - effect increases with curvature (as f/D decreases)
- Symmetry
    – on-axis systems see linear cross-polarization
    – off-axis feeds introduce asymmetries & R/L squint -> uncertainties on Stokes V
- Feedhorn & Polarizers
    –introduce further effects (e.g. “leakage”)

Note: Optical effects are the same in linear/circular basis, but response to the electronics effects is different

Conming soon - EAS 2025

EAS 2025 (23-27 June 2025, Cork, Ireland)

Special Session

Polarimetry of young stellar objects: instrumentation, observations and models (including laboratory experiments)

https://eas.unige.ch/EAS2025/session.jsp?id=SS20

Deadline for abstract submission: 3 March 2025

Deadline for early bird registration: 28 April 2025

Deadline for regular registration: 22 Jun 2025

Summary

- Polarimetry is not straightforward
  - Leakages are not negligible wrt the quantities we need to measure
  - For highly polarized sources or highly polarized antennas, one needs to derive absolute leakages (physically rotate the feed or antenna)
  - Constraint on the observing tracks (need enough parallactic angle coverage for calibration + phase stability)
  - Effects like leakages and beam squint are more severe with increasing distance from the center-> wide field polarimetry is challenging

IPAG
Institut de Planétologie
et d'Astrophysique
de Grenoble

European Research Council
Established by the European Commission

Thank you

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References

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